THE CHAOTIC COST-PLUS PRICING MODEL

Vesna D. Jablanovic
Associate Professor of Economics , University of Belgrade, Serbia [1]
vesnajab <vesnajab@ptt.rs>

ABSTRACT

In this paper, it is examined one pricing practice often used by oligopolistic firm: cost-plus pricing. Cost-plus pricing refers to the setting of a price equal to average variable cost plus a markup. The pricing of airline tickets illustrates this concept presented in this paper as it is applied in a real-world oligopolistic market. It is important concept because the trend toward the formation of global oligopolies has accelerated as the world's answer on the current financial crise. The basic aim of this paper is to construct a relatively simple chaotic cost-plus pricing model that is capable of generating stable equilibria, cycles, or chaos.

A key hypothesis of this work is based on the idea that the coefficient, $\pi = \frac{d}{(r-1)\, b \left(1+\frac{1}{e}\right)}$ *, plays a crucial role in explaining local stability of the oligopolistic firm's output, where, d – the coefficient of the average variable cost function of the oligopolistic firm, b - the coefficient of the inverse demand function, r - the coefficient of price growth.*

Keywords: *oligopolistic firm , cost-plus pricing, growth, output, chaos*

1. INTRODUCTION

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983, 1997) , Grandmont (1985), Goodwin (1990), Medio (1993,1996), Lorenz (1993), Jablanovic (2010, 2011, 2012), among many others.

The basic aim of this paper is to construct a relatively simple chaotic cost-plus pricing model that is capable of generating stable equilibria, cycles, or chaos Cost-plus pricing is a pricing method in which the selling price is established by adding a markup to total variable costs. Variable cost-plus pricing is not suitable in situations where fixed costs are a major component of total costs.

For example, assume total variable costs for producing one unit of a product are $10 and a markup of 60% is added. The selling price as determined by this variable cost-plus pricing method would be $16. If contribution to fixed costs per unit is estimated at $4, then profit per unit would be $2.

In the real world, firms often lack exact information to determine the price that maximizes their profits. In these cases, firms usually use cost-plus pricing. In this paper, the firm estimates the average variable cost for a »normal« output level and then adds a certain percentage or markup over average variable cost to determine the price of the commodity. The markup is set sufficiently high to cover average variable and fixed costs and provide a profit margin for the oligopolistic firm. The markup varies depending on the industry and demand conditions. For example, the markup is higher for products facing less elastic demand or in periods of high demand.

[1] Faculty of Agriculture, Nemanjina 6, Belgrade, Serbia, vesnajab@ptt.rs

2. THE MODEL

Cost-plus pricing is fairly common in oligopolistic industries, and, under certain conditions, it is not inconsistent with profit maximization. That is, to the extent that the markup is varied inversely with the elasticity of demand of the product, it leads to a price which is approximately the profit-maximizing price. This can be shown as follows:

$$m = \frac{P - AVC}{AVC} \qquad (1)$$

where P is price, AVC is average variable cost, and m is the markup over AVC, expressed as a percentage of AVC.

Solving for P, we obtain

$$P = AVC\,(1+m) \qquad (2)$$

On the other hand, marginal revenue is

$$MR_t = P_t\left[1 + \left(\frac{1}{e}\right)\right] \qquad (3)$$

Where MR – marginal revenue; P –price; e – the coefficient of the price elasticity of demand. Solving for P we get

$$P_t = \frac{MR_t}{\left(1 + \frac{1}{e}\right)} \qquad (4)$$

Since profits are maximized where MR =MC, it is possible to substitute MC for MR in the above formula and get

$$P_t = \frac{MC_t}{\left(1 + \frac{1}{e}\right)} \qquad (5)$$

To the extent that the firm's MC is constant over a wide range of outputs, MC = AVC. Substituting AVC for MC in the above formula , we get

$$P_t = \frac{AVC_t}{\left(1 + \frac{1}{e}\right)} \qquad (6)$$

Above formula for profit maximization equals formula for the markup, if 1+m=1/[1+(1/e)] or if m= -1/(e+1). Thus the oligopolistic firm will maximize profits if its markup is inversely related to the price elasticity of demand for the commodity. For example, when e= -3, m should be ½, or 50%. This means that if AVC=$100, P should equal $150 (so that the markup is 50% of AVC) for the firm to cover all costs and maximize profits.

Using cost-plus pricing with a markup that varies inversely with the price elasticity of demand is consistent with profit maximization. Cost-plus pricing is one of many rules of thumb that firms are forced to use in the real economic world .

Given industry demand of

$$P_t = a - b\,Q_t \qquad (7)$$

Where P – oligopolistic firm's price; Q – oligopolist's output; a, b – coefficients of the inverse demand function.

If it is supposed that

$P_{t+1} = P_t + r P_{t+1}$ (8)

then

$Pt = (1-r)(a-b Q_{t+1})$ (9)

Further, if it is supposed that a=0, then

$Pt = (r-1) b Q_{t+1}$ (10)

Average variable cost function, AVC_t is expressed as

$AVC_t = c + d Q_t + f Q_t^2$ (11)

Where AVC – average variable cost; Q – oligopolist's output ; c , d , f – coefficients of the quadratic average variable cost function.

It is supposed that a=0 and c=0. Substitution (10) and (11) in (6) gives

$$(r-1)\, b\, Q_{t+1} = \frac{d\, Q_t + f\, Q_t^2}{\left(1+\frac{1}{e}\right)} \quad (12)$$

By substitution one derives:

$$Q_{t+1} = \frac{d}{(r-1)\, b \left(1+\frac{1}{e}\right)} Q_t - \frac{f}{(1-r)\, b\left(1+\frac{1}{e}\right)} Q_t^2 \quad (13)$$

Further, it is assumed that the oligopolistic firm's output is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the oligopolistic firm's output growth rate depends on the current size of the oligopolistic firm's output , Q, relative to its maximal size in its time series Q^m. We introduce q as $q = Q / Q^m$. Thus q range between 0 and 1. Again we index q by t, i.e., write q_t to refer to the size at time steps t = 0,1,2,3,... Now , growth rate of the oligopolistic firm's output is measured as

$$q_{t+1} = \frac{d}{(r-1)\, b \left(1+\frac{1}{e}\right)} q_t - \frac{f}{(1-r)\, b\left(1+\frac{1}{e}\right)} q_t^2 \quad (14)$$

This model given by equation (14) is called the logistic model. For most choices of α , b, d, f, and e there is no explicit solution for (14). Namely, knowing α , b, d, f and e and measuring q_0 would not suffice to predict q_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (14) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of q_t . This difference equation (14) will posses a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point q_0 the solution is highly sensitive to variations of the parameters α , b, d, f and e; secondly, given the parameters α , b, d, f and e the solution is highly sensitive to variations of the initial point q_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

3. THE LOGISTIC EQUATION
The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0,4], \quad z_t \in [0,1] \qquad (15)$$

is equivalent to the iteration of growth model (14) when we use the following identification:

$$z_t = \frac{(r-1) f}{(1-r) d} q_t \qquad (16)$$

and

$$\pi = \frac{d}{(r-1) b \left(1 + \frac{1}{e}\right)} \qquad (17)$$

Using (14), and (16) we obtain

$$z_{t+1} = \frac{(r-1) f}{(1-r) d} q_{t+1} = \frac{(r-1) f}{(1-r) d} \left[\frac{d}{(r-1) b \left(1+\frac{1}{e}\right)} q_t - \frac{f}{(1-r) b \left(1+\frac{1}{e}\right)} q_t^2 \right] =$$

$$= \frac{f}{(1-r)\, b \left(1+\frac{1}{e}\right)} q_t - \frac{(r-1) f^2}{(1-r)^2\, b\, d \left(1+\frac{1}{e}\right)} q_t^2$$

On the other hand, using (15), and (17) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = \left[\frac{d}{(r-1) b \left(1+\frac{1}{e}\right)} \right] \left[\frac{(r-1) f}{(1-r) d} \right] q_t \left\{ 1 - \left[\frac{(r-1) f}{(1-r) d} \right] q_t \right\}$$

$$= \frac{f}{(1-r)\, b \left(1+\frac{1}{e}\right)} q_t - \frac{(r-1) f^2}{(1-r)^2\, b\, d \left(1+\frac{1}{e}\right)} q_t^2$$

Thus we have that iterating $q_{t+1} = \frac{d}{b(r-1)\left(1+\frac{1}{e}\right)} q_t - \frac{f}{b\,(1-r)\left(1+\frac{1}{e}\right)} q_t^2$ is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{(r-1) f}{(1-r) d} q_t$ and $\pi = \frac{d}{(r-1) b \left(1+\frac{1}{e}\right)}$.

It is important because the dynamic properties of the logistic equation (15) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that : (i) For parameter values $0 < \pi < 1$ all solutions will converge to z = 0; (ii) For $1 < \pi < 3,57$ there exist fixed points the number of which depends on π; (iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$; (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$; (v) For $3 < \pi < 4$ all solutions will continously fluctuate; (vi) For $3,57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

4. CONCLUSION

This paper suggests conclusion for the use of the simple chaotic cost-plus pricing model of a profit – maximizing the oligopolistic firm in predicting the oligopolistic firm's output. The model (14) has to rely on specified parameters α , b, d, f and e , and initial value of the oligopolistic firm's output, q_0. But even slight deviations from the values of parameters α , b, d, f and e , and initial value of the oligopolistic firm's output, show the difficulty of predicting a long-term the oligopolistic firm's, q_0.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \dfrac{d}{(r-1)\,b\left(1+\dfrac{1}{e}\right)}$ plays a crucial role in explaining local stability of the oligopolistic firm's output where, d – the coefficient of the average variable cost function of the oligopolistic firm , b - the coefficient of the inverse demand function, α - the growth coefficient of the price. The quadratic form of the average variable cost function of the oligopolistic firm is important ingredient of the presented chaotic cost-plus pricing model (14).

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